and positioning based on the RSSI algorithm with respect to environmental condition. The main requirements' specification, motivation and their relevant features is discussed in chapter 5.

3.3 Validity Evaluation

Before doing the mapping study, we gained enough knowledge about the RSSI context from the company to reduce the threat of internal validity on the SMS. In this case, we considered firstly our applied keywords in the search string. We utilized the "RSSI" keyword as an indicator in our search string. Secondly, in our primary study, we focused on three main databases: IEEE, Engineering Village and Scopus which have the most important papers in the field of software engineering. So, we had enough relevant papers to analyze. Finally, we studied all abstracts, titles and keywords of the selected papers for keywording with respect to our accepted research questions. In cases which abstracts were not clear and informative enough, we studied introduction and conclusion parts. However, there is a risk of judgmental error since we could not read the papers completely and all the evaluation has been done by one person. Also, in the conclusion part we only focused on the RSSI-based localization and specified the lack of research in different areas of this context.

In our experiments, although we tried to run the experiments in a well and controlled situation (design of the experiments, variable measurements …), there were parameters that can affect our experiments' results. The target node had a battery whose voltage was not always enough, although we renewed the battery constantly. All the experiments have been done by a 12 volt power supply and we did not test the other voltages. The sampling times were sometimes too short and with respect to the RSSI characteristic, they can affect the results. We only had three sensor nodes (the minimum number of anchors for doing localization experiment) while we knew how effective is anchor density in this context. Moreover, our sensors did not have suitable antennas (however, antenna's type has a considerable effect on the experiments' results). Finally, as we mentioned in chapter 2, people's movement has clear effect on the accuracy of the RSSI sample, but in rare experiments we had uncontrolled and unintentionally office staff's movement.

In the literature review phase, firstly we focused on the papers that the company specified for the author. All these papers were from valid sources, peer reviewed (journals and conferences), related to our purpose of the research and in the scope of RSSI-based localization. Based on the gained knowledge at the first LR, the author became familiar with the context, the research background and related work. In the second LR, the focus was mostly on learning the significant concepts and theories, new solutions and algorithms, and utilizing the ideas proposed by the literature in our software (defining new requirement). In addition, all the applied literature in the second LR was peer reviewed.

Chapter 4

Localization Process

*The aim of this chapter is to present practical steps of implementing a localization system. The content is based on the literature review. The introduction (4.1) encompasses (Baunach et al., 2007) localization parameters, simplified measurement formulas which were studied in chapter 2 (background), different localization approaches and considering antennas' types based on the classification of (*Hamdoun *et al., 2014). Section (4.2) considers localization steps with focus on channel identification and its effects on mathematical formulas of measurement, Path loss exponent calculation and the RSSI optimization. Section (4.3) studies positioning algorithm with respect to solving linear equation and triangle centroid location algorithm presented by (Jungang Zheng et al., 2010). Section (4.4) considers an innovative positioning algorithm proposed by (Zhang et al., 2011) to reduce the ranging error. Finally, section (4.5) presents a metric for the localization accuracy.*

4.1 Introduction

(Baunach et al., 2007) identify eight parameters to describe localization systems:

- The localization of objects in the system can be relative to each other or absolute (based on anchors with known position)
- The process of localization can be done periodically or based on specific requirement (occasionally)
- The initiator of the process can be target node or the anchor nodes
- The localization approach can be active (the surrounding objects determine the location of target node), passive (target node determines its location) or interactive (combination of the mentioned approaches)
- The implementation algorithm can be two dimensional or more
- The localization system can be fast to track moving object just position static objects
- The anchors in the system can be tightly coupled (wired to the central unit) or loosely coupled (with wireless communication)
- The system can be centralized (with a central unit for measurement and positioning) or decentralized (with considering the network traffic management)

In the RSSI-based localization, as mentioned in chapter 2, distance measurement is a significant phase and path loss shadowing model is used as the mathematical algorithm for measuring distance. Based on the path loss shadowing model and in the initial phase (calibration phase) path-loss exponent (to adaptive the system with the environment) and path loss offset (measured in 1 meter reference distance) are calculated. Then with calculated parameters (path loss exponent is the propagation coefficient parameter) the distance is calculated. The following formulas explain the method (discussed in chapter 2) (Fink & Beikirch, 2009; Chongburee et al., 2009).

$$\text{Path loss shadowing mode: } P_L(d) = P_L(d_0) + 10\beta \log\left(\frac{d}{d_0}\right) + X_\sigma \rightarrow$$

$$\text{Path loss exponent parameter when } d_0 \text{ is 1m: } \beta = \frac{P_L(d) - P_L(d_0) - X_\sigma}{10 \log(d)}$$

$$\text{Distance: } d = 10^{\left(\frac{P_L(d) - P_L(d_0) - X_\sigma}{10\,\beta}\right)}$$

Xσ (zero-mean Gaussian random variable with standard deviation σ) in the above formulas indicate the shadowing effect. Multipath fading because of objects that obstructing the signal propagation path between transmitter and receiver causes this effect. Totally, shadow fading can be studied in two different types; path loss dependent and path loss independent (out of the context of this study) shadow fading. Path loss dependent shadow fading is the residual error when we fit path loss model $(A + m\log(d))$ to the measurement. Since this kind of shadow fading depends on path loss law model and the method of fitting, therefore different path loss model have different shadow fading results (Salo et al., 2005). In this study and in the experiment phase, because of simplicity, we disregard *Xσ* parameter in the above formulas and apply following formulas to calculate the path loss exponent and distance.

$$\beta = \frac{P_L(d) - P_L(d_0)}{10 \log(d)}, \quad d = 10^{\left(\frac{P_L(d) - P_L(d_0)}{10\,\beta}\right)}$$

Basically, localization approach has two phases: distance or angle measurement (based on mentioned formula) and distance or angle combination (using geometrical principles). In the combining phase hyperbolic Multilateration (this technique is named trilateration when three reference nodes is used. Positioning by finding the intersection of three circles around each reference node, figure 19(a)), triangulation (positioning by applying trigonometry laws in methods such as AoA, figure 19(b)) and maximum likelihood (positioning by minimizing the differences between measured and estimated distances, figure 19(c)) are three popular methods (Pal, 2010).

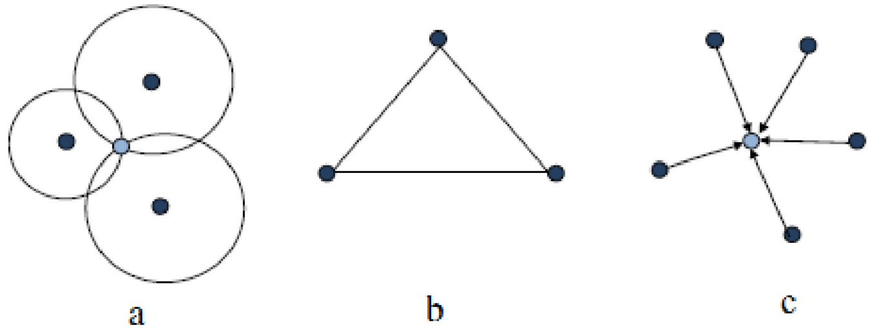


Figure 19: Three different methods in the localization combining phase (Pal, 2010)

The accuracy is a significant metric in the context of localization and different methods and techniques tries to improve the accuracy. As well as the different techniques, hardware and specifically antenna type has considerable effect on position accuracy in localization. (Hamdoun et al., 2014) focus and compare different antenna systems and different diversity combining techniques to evaluate the effect of antenna type on the reliability of wireless link and localization performance. The reason of their research is the distance estimation with RSSI measurement in an indoor situation is effected by propagation environment. The effect of using multiple antennas in three system communication models was evaluated (Hamdoun et al., 2014).

- (SIMO) Single Input Multiple Output: the transmitter has single antenna and receiver has multiple antennas (figure 20(a)). Therefore, the receiver can mitigate the fading effects by receiving N independent copies of the transmitted signal.
- (MISO) Multiple Input Single Output: the transmitter has multiple antennas and receiver has single antenna (figure 20(b)). In comparison with SIMO model, the process is done in transmitter instead of receiver.
- (MIMO) Multiple Input Multiple Output: in this model both receiver and transmitter have multiple antennas (figure 20(c)) (Hamdoun et al., 2014).

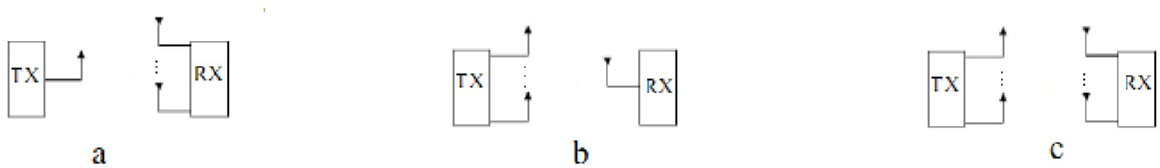


Figure 20: SIMO (a), MISO (b) and MIMO (c) systems (Hamdoun et al., 2014)

Due to apply multiple antennas in the receivers (Hamdoun et al., 2014) applied three diversity combining techniques and then the RSSI values were used in the distance measurement.

- SC (Selecting combining method) which select maximum RSSI value among the N receiver's antennas ($RSSI_{max} = \max\{RSSI_1, \dots, RSSI_N\}$).
- EGC (Equal Gain Combining method) which averages between received RSSI values ($RSSI_{avg} = \frac{1}{N}\sum_{i=1}^{N} RSSI_i$).

- MRC (Maximum Ratio Combining method) with ($RSSI_{mrc} = \frac{1}{\sum_{i=1}^{N} RSSI_i} \sum_{i=1}^{N} RSSI_i^2$).) formula (Hamdoun et al., 2014).

Figure 21 demonstrate localization based on trilateration with applying different antenna models.

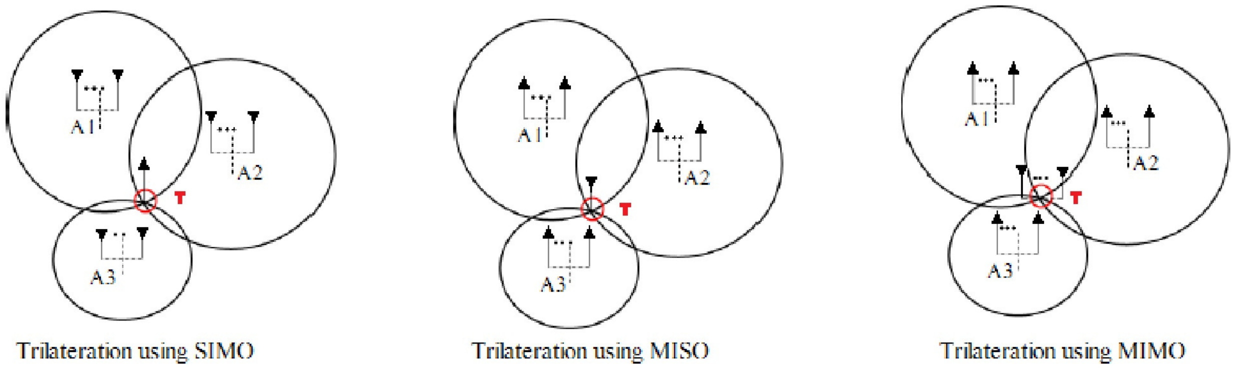


Figure 21: Trilateration method using SIMO, MISO and MIMO antenna model (Hamdoun et al., 2014)

Based on the results SISO (Single Input Single Output antenna) has the worst and MIMO has the best localization accuracy. Although increasing the number of antennas improves the accuracy, it can increase the complexity of system. Also they explain that among different diversity combining methods, MRC has the best performance (Hamdoun et al., 2014).

4.2 Localization Steps

Based on (Chuan-Chin Pu et al., 2011) location tracking system includes three steps: signal and information processing, realization of the system by implementing different techniques and finally storing, analyzing, monitoring and displaying the relevant localization information in a centralized server. The third step is defined in chapter 5. Data mining and signal processing form the first task (information handling) in the localization system design. Moving from raw RSSI values to find location coordinate has some steps (Fink & Beikirch, 2009) that figure 22 can illustrate them.

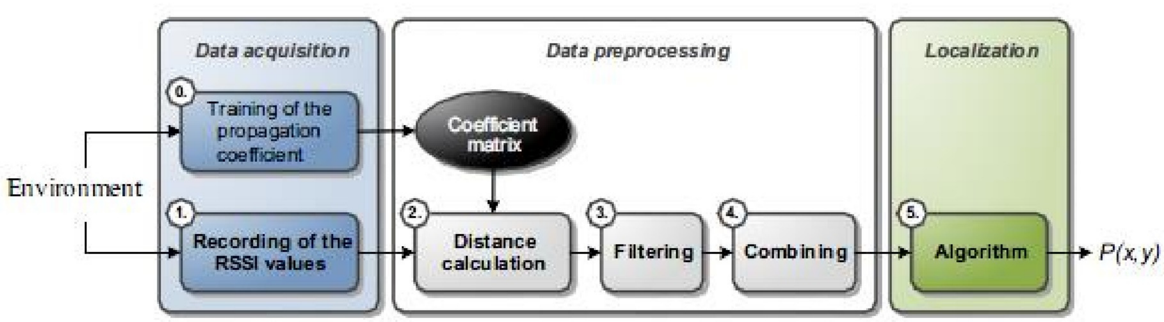


Figure 22: Structure of localization system (Fink & Beikirch, 2009)

Firstly, in the data acquisition phase, RSSI values are collected from the anchors. These RSSIs are used to find the suitable environmental parameters (system calibration). Theses environmental parameters are fixed during the localization unless considerable changes occur in the environment. After calibration the anchors receive continuous RSSIs from the target nodes and then with both environmental parameters and RSSI values (and using techniques to RSSI signal improvement, optimization, filtering) we can measure distances by using path loss model. Finally, the distances between anchors and target nodes are used by geometrical techniques such as trilateration to find the position of the target sensor node in an environment (Fink & Beikirch, 2009).

Since this technique uses RSSI values to distance estimation, the level of RSSI is significant. Based on the RSSI characteristics and models that mentioned in the previous chapters, these values of measuring are very volatile with variation in indoor environment. The reasons of this variation can be categorized in three models (C. C. Pu et al., 2012):

- *"Small scale multipath fading (small-scale fading explains the rapid fluctuation of received power because of small sub wavelength changes in the receiver position. This effect occurs because of constructive and destructive interference of multipath waves.)*
- *Medium scale shadowing model* (because of different obstacles) and
- *Large scale path loss model (path loss: the difference between transmitter and receiver power)"* (C. C. Pu et al., 2012).

Therefore, different methods in the different steps of localization try to improve the accuracy of RSSI ranging. RSSI signal improvement concentrate on noise and small scale fading to increase stability of the RSSI signal (C. C. Pu et al., 2012).

In the calibration phase (environmental characterization) defining the suitable parameters is significant since the accurate value of the path loss exponent (β) has important effect on precision of the distance and consequently the localization accuracy. In the path loss exponent measurement waves' reflection can bring down the accuracy. Therefore, channel identification (identifying the receive data from transmitter is LOS (Line of Sight) or NLOS (Non-line of Sight)) is important to mitigate this effect.

Channel Identification:

Propagation condition in a wireless communication is divided into LOS and NLOS channels and affects the accuracy of the moving object positioning. If the propagation is LOS between receiver and transmitter we can achieve high accuracy in localization, however in an indoor places we have different obstacles and people movement which can block the path between anchors and target node (receiver and transmitter). These obstacles can cause NLOS error (because of signal reflection and diffraction). If the measurement of more than three anchors are available and at least three of them are LOS, the range measurements of NLOS anchor nodes can be ignored in moving object positioning. However, if there are the measurements of only three anchor nodes and one of them is NLOS, some algorithm such as Range Scaling Algorithms (RSA) can be used to alleviate the NLOS error (Venkatraman & Caffery, 2002; Wang et al., 2013; Kegen Yu et al., 2009).

There are different techniques for localization algorithm to improve their tolerance in NLOS condition. These techniques are divided into two categories. First category includes techniques that intend to identify NLOS channel to mitigate its negative effects on localization. The second category includes techniques that directly alleviate the NLOS error in range or location estimations. Since there is a mutually exclusive relationship between LOS and NLOS, the channel identification is a binary hypothesis test. Considering, hypothesis testing for LOS/NLOS applies probability distribution and compares NLOS and LOS with each other it is necessary to consider time consumption and complexity factors (Venkatraman & Caffery, 2002; Wang et al., 2013; Xiao et al., 2013). In the hypothesis testing approach (Xiao et al., 2013; Wann & Chin, 2007) describe hypotheses for LOS/NLOS based on:

$$H_l: \alpha \leq \alpha_t \;\; \rightarrow LOS\ conditions$$

$$H_n: \alpha > \alpha_t \;\; \rightarrow NLOS\ conditions$$

Then to identify the correct channel, they determine specific function for "α" and a threshold for "α_t". Therefore, the distance estimation model (log-normal shadowing model) based on LOS/NLOS is:

$$P_L(d) = P_L(d_0) + 10\beta_{los/nlos} \log\left(\frac{d}{d_0}\right) + \; X_{\sigma(los/nlos)}$$

Based on the channel identification and above formula, the distance measurement and path loss exponent calculate differently in LOS or NLOS conditions.

(Kegen Yu et al., 2009) studied LOS/NLOS identification based on the difference probability distribution between these two channels. They used the Generalized Likelihood Ratio Test (GLRT) for identification and choosing one the hypotheses when NLOS error has deterministic mean and variance. Their hypothesis testing is based on knowing the LOS and NLOS probability in advance otherwise the other tests should be selected. Assume that in an experiment *L* distance measurement sample is available and each distance sample is sum of true distance and distance measurement error:

$$r = [r_1 \quad r_2 \, \dots \, r_L]^T$$

$$r_i = d + \varepsilon_i$$

Where d is true distance and ε_i is error distance. $p(r|\, d, H_l)$ and $p(r|\, d, H_n)$ as the conditional probability density function (PDF) of r under LOS (H_l) and NLOS (H_n) hypotheses are respectively (Kegen Yu et al., 2009):

$$p(r|d, H_l) = \frac{1}{\sqrt{2\pi}\sigma_{los}^L} \exp\{-\frac{1}{2\sigma_{los}^2}\sum_{i=1}^{L}[r_i - (\mu_{los} + d)]^2\}$$

$$p(r|d, \mu_{nlos}, \sigma_{nlos}, H_n) = \frac{1}{\sqrt{2\pi}\sigma_{nlos}^L} \exp\left\{-\frac{1}{2\sigma_{nlos}^2}\sum_{i=1}^{L}[r_i - (\mu_{nlos} + d)]^2\right\}$$

Where μ_{los} is the mean and σ^2_{los} is the variance of ε_i in the LOS condition and μ_{nlos} is the mean and σ^2_{nlos} is the variance of ε_i in the NLOS condition. Therefore, based on GLRT, H_n (NLOS hypothesis) is decided if:

$$A(r) = \frac{P(r|\bar{d}_{nlos}, \bar{\mu}_{nlos}, \bar{\sigma}_{nlos}, H_n)}{P(r|\bar{d}_{los}, H_l)} > \frac{P(H_l)}{P(H_n)}$$

Where $\bar{d}_{los}$ and $\bar{d}_{nlos}$ are the maximum likelihood estimates of the unknown LOS and NLOS distances, $\bar{\mu}_{nlos}$ and $\bar{\sigma}_{nlos}$ are unknown noise mean and unknown noise standard deviation, $P(H_l)$ and $P(H_n)$ are the known prior probability of NLOS and LOS conditions (Kegen Yu et al., 2009).

As mentioned before, in the system calibration step (environmental characterization) we intend to determine two parameters; the received power at the reference distance d_0 ($P_L(d_0)$) and path loss exponent (β coefficient in the log normal shadowing model) which is highly dependent on the environment of experiment. The path loss exponent can be calculated empirically by doing M times of measurement. (Y. Chen et al., 2012) calculated (β) value based on the path-loss log normal model ($RSSI_d = -(10\beta \log d) + A$) by applying the Least Square Method as follow:

Path Loss Exponent Calculation (Y. Chen et al., 2012)***:***

$$\begin{cases} RSSI_1 = -(10\beta \log d_1) + A \\ RSSI_2 = -(10\beta \log d_2) + A \\ \vdots \\ RSSI_m = -(10\beta \log d_m) + A \end{cases}$$

After M times measurement we subtract the first equation from the other equations, so we have:

$$\begin{cases} RSSI_2 - RSSI_1 = -\left(10\beta \log \frac{d_2}{d_1}\right) \\ RSSI_3 - RSSI_1 = -\left(10\beta \log \frac{d_3}{d_1}\right) \\ \vdots \\ RSSI_m - RSSI_1 = -\left(10\beta \log \frac{d_m}{d_1}\right) \end{cases}$$

We can write the above equations in the form of a matrix like ($CX = R$) where X = [β], C and R are respectively:

$$C = \begin{bmatrix} -10\log(\frac{d_2}{d_1}) \\ -10\log(\frac{d_3}{d_1}) \\ \vdots \\ -10\log(\frac{d_m}{d_1}) \end{bmatrix} \quad , \quad R = \begin{bmatrix} RSSI_2 - RSSI_1 \\ RSSI_3 - RSSI_1 \\ \vdots \\ RSSI_m - RSSI_1 \end{bmatrix}$$

To solve the mentioned linear equation system by Least Square method, $\|CX - R\|$ is minimized when $C^TCX = C^TR$. Therefore, $X = (C^TC)^{-1}C^TR$ (Y. Chen et al., 2012).

Since there are different factors such as temperature, multipath effects, non-line of sight effect and so on, the propagation of wireless signal is random. Therefore, it is significant to filter the current received RSSI values before substituting into the formula and calculate the distance. In the way of optimizing RSSI values, average statistical model is not always effective for large disturbance (Zhu Minghui & Zhang Huiqing, 2010). Gaussian filter is the model can be used in this step to improve the accuracy of localization. Since, in this model we can select the RSSI values in the high probability areas and then we can apply the mean filter for the optimized RSSI values (Zhu Minghui & Zhang Huiqing, 2010; Qingxin Zhang et al., 2010).

RSSI Value Optimization (Zhu Minghui & Zhang Huiqing, 2010)***:***

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(x-\mu)^2}{2\sigma^2}} \quad gaussian\ model$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \quad , \qquad \sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2$$

The received RSSI values should be selected in this range: $0.6 \leq F(x) \leq 1$ where 0.6 is calculated based on the experience of value engineering. (Zhu Minghui & Zhang Huiqing, 2010) show that the RSSI range in this approach of optimization is

$$[0.15\sigma + \mu, 3.09\sigma + \mu]$$

Where σ and μ are respectively:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(RSSI_i - \frac{1}{n}\sum_{i=1}^{n} RSSI_i)^2} \quad , \qquad \mu = \frac{1}{n}\sum_{i=1}^{n} RSSI_i$$

4.3 Description of the Positioning Algorithm

The positioning algorithm is used to calculate the coordinates of the target node. In two dimensions localization the number of anchor nodes should be at least three (Deng et al., 2008). Maximum likelihood estimation method can be used in this step. In this method we should know the position of the anchors (reference nodes) as (x_1, y_1), (x_2, y_2), …, (x_n, y_n), and their distances from the target node, which is calculated by the log-normal shadowing model, $d_1, d_2, \ldots, d_3$. If we assume the coordinate of the target node as (x, y), then the following non-linear equations in two-dimensional space exist (Deng et al., 2008):

$$\begin{cases} (x-x_1)^2+(y-y_1)^2=d_1^2 \\ (x-x_2)^2+(y-y_2)^2=d_2^2 \\ \vdots \\ (x-x_n)^2+(y-y_n)^2+d_n^2 \end{cases}$$

Then the last equation is subtracted from the other equations as (Deng et al., 2008):

$$\begin{cases} x_1^2-x_n^2-2(x_1-x_n)x+y_1^2-y_n^2-2(y_1-y_n)y=d_1^2-d_n^2 \\ \vdots \\ x_{n-1}^2-x_n^2-2(x_{n-1}-x_n)x+y_{n-1}^2-y_n^2-2(y_{n-1}-y_n)y=d_{n-1}^2-d_n^2 \end{cases}$$

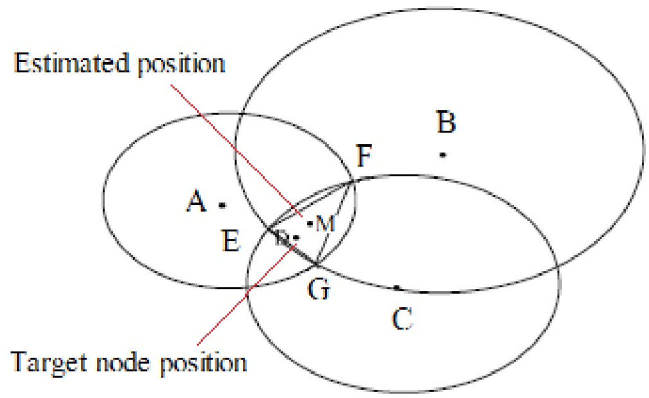
Now there is a linear equation that can be demonstrated as *AX=b,* where *A*, *b*, and *X* are respectively (Deng et al., 2008):

$$A=\begin{bmatrix} 2(x_1-x_n) & 2(y_1-y_n) \\ & \vdots \\ 2(x_{n-1}-x_n) & 2(y_{n-1}-y_n) \end{bmatrix},$$

$$b=\begin{bmatrix} x_1^2-x_n^2+y_1^2-y_n^2-d_1^2+d_n^2 \\ \vdots \\ x_{n-1}^2-x_n^2+y_{n-1}^2-y_n^2-d_{n-1}^2+d_n^2 \end{bmatrix}, \qquad X=\begin{bmatrix} x \\ y \end{bmatrix}$$

The coordinate of the target node can be given by estimation method for standard minimum mean square by: $\hat{X}=(A^TA)^{-1}A^Tb$ (Deng et al., 2008).

(Jungang Zheng et al., 2010) studied another model (triangle centroid location algorithm) to estimate the position of the target node which is based on calculation the center of triangle area of the target node. In this approach (for three anchor) anchor nodes *(A, B, C)* form circle areas that the estimated distances between the anchors and target node (r_A, r_B, r_C) are their radii. The overlapping area of the anchors' circles makes three points which are vertices of a triangle. This area is "target node triangle area" and the center of this triangle is the target node's coordinate (figure 23).



$$\begin{cases} x_m = \dfrac{x_e + x_f + x_g}{3} \\ y_m = \dfrac{y_e + y_f + y_g}{3} \end{cases}$$

Figure 23: Triangle centroid localization model (Jungang Zheng et al., 2010)

(Yingxi et al., 2012) based on the geometrical theory calculated the position of the target node in a randomly distributed sensor network area in R3 Space. The following is the defined vector set for all sensor nodes (n) in the system:

$$X = (x_1, y_1, z_1, x_2, y_2, z_2, \dots, x_n, y_n, z_n)$$

4.4 Adjacent Correction Positioning Algorithm by (Zhang et al.)

(Zhang et al., 2011) proposed an adjacent correction positioning algorithm based on multilateral positioning (applying ML estimation) to reduce unilateral ranging error. Their algorithm improves the accuracy and stability of the localization system. In this approach the anchors send their RSSI values (instead of previous approaches which anchors were receiver) and other network information (their ID and coordinates) to the target node and correction node. The idea of applying the adjacent correction node is to find the correction factors, discrimination coefficient and measure the error between anchor and correction node. The correction node moves to different places on a circle that the target node is in the center. In fact, in this approach we measure the real distance between anchors and correction node in the initialization of network. Then we calculate the mentioned factors based on the real measured distances to use in the target node localization. Figure 24 demonstrates this algorithm where there are eight anchors ($R_n(x_n, y_n)$), one target node ($B(x, y)$) and one correction node ($C(\Delta x, \Delta y)$). The following concept is defined to describe the algorithm (Zhang et al., 2011):

- The actual distance between anchor R_n and correction node is $d_{\Delta n}$.
- The measured distance between anchor R_n and correction node is $d'_{\Delta n}$.
- The measured distance between anchor R_n and target node is d'_n.
- The distance between anchor R_n and target node after correction is d_n.

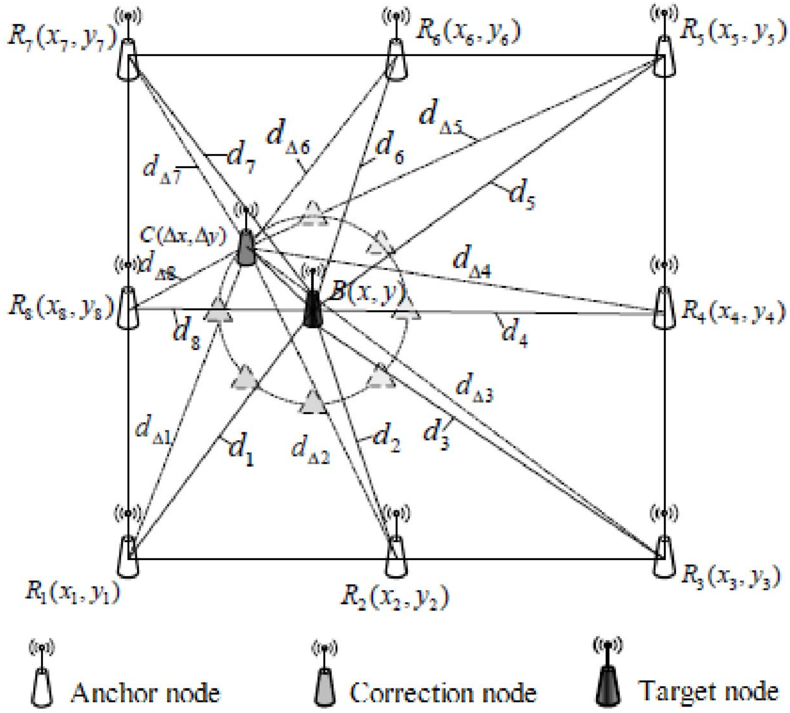


Figure 24: Adjacent correction algorithm (Zhang et al., 2011)

In the positioning step, the correction factor for eight anchors (η) and discrimination coefficient between anchor and target node (μ_n) are defined as:

$$\eta = \left(\frac{d'_{\Delta 1} - d_{\Delta 1}}{d'_{\Delta 1}}\right) + \left(\frac{d'_{\Delta 2} - d_{\Delta 2}}{d'_{\Delta 2}}\right) + \cdots + \left(\frac{d'_{\Delta 8} - d_{\Delta 8}}{d'_{\Delta 8}}\right) = 8 - \sum_{n=1}^{8} \frac{d_{\Delta n}}{d'_{\Delta n}}$$

$$\mu_n = \lambda e^{1 - \frac{d'_n}{d'_{\Delta n}(1-\eta)}}$$

Where λ numeric area is (0, 1) and is based on environment test at the initialization deployment. Now the range error between anchor R_n and correction node is:

$$\varepsilon_n = d'_{\Delta n} - d_{\Delta n}$$

Finally the corrected distance between R_n and target node is:

$$d_n = d'_n - \mu_n \varepsilon_n$$

Now we can use the corrected distance (d_n) in the multilateration positioning algorithm (Zhang et al., 2011) mentioned in Section (4.3).

4.5 Localization Accuracy Metric

In the context of localization accuracy (Wang et al., 2012) presented a metric based on the localization error (LE) between the estimated position and the actual position of the target node. In a two-dimensional positioning it is supposed that there are N anchors and a target node with the actual coordinate (X, Y) and estimated coordinate (X_i, Y_i). Accordingly we have:

$$LE(X_i) = (X - X_i)\,, \qquad LE(Y_i) = (Y - Y_i)\,, \qquad i = 1, 2, \dots, N$$

Then the mean of positioning error is (Wang et al., 2012):

$$ME(X) = \sum_{i=1}^{N} \frac{LE(X_i)}{N}\,, \qquad ME(Y) = \sum_{i=1}^{N} \frac{LE(Y_i)}{N}$$

And the error variance is defined as:

$$\Delta X_i = LE(X_i) - ME(X), \qquad \sigma_{Xi}^2 = (\Delta X_i)^2$$
$$\Delta Y_i = LE(Y_i) - ME(Y), \qquad \sigma_{Yi}^2 = (\Delta Y_i)^2$$

$$\sigma_{XY}^2 = \frac{\sum_{i=1}^{N} \sigma_{Xi}^2 + \sum_{i=1}^{N} \sigma_{Yi}^2}{N}$$

Finally the standard deviation *SD (σ)* gives us the average lower bound variance of the target node positioning error (Wang et al., 2012):

$$SD(\sigma) = \sqrt{\sigma_{XY}^2}$$